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Existence of solutions for perturbed abstract measure functional differential equations

Xiaojun Wan and Jitao Sun*

* Correspondence: sunjt@sh163.net
Department of Mathematics, Tongji University, Shanghai 200092, China

Abstract

In this article, we investigate existence of solutions for perturbed abstract measure functional differential equations. Based on the Arzelà-Ascoli theorem and the fixed point theorem, we give sufficient conditions for existence of solutions for a class of perturbed abstract measure functional differential equations. Our system includes the systems studied in some previous articles as special cases and our sufficient conditions for existence of solutions are less conservative. An example is given to illustrate the effectiveness of our existence theorem of solutions.

1 Introduction

Abstract measure differential equations are more general than difference equations, differential equations, and differential equations with impulses. The study of abstract measure differential equations was initiated by Sharma [1] in 1970s. From then on, properties of abstract measure differential equations have been researched by various authors. But up to now, there were only some limited results on abstract measure differential equations can be found, such as existence [2-6], uniqueness [2,3,5], and extremal solutions [3,4,6]. There were also several researches on abstract measure integro-differential equations [7,8]. The study on abstract measure differential equations is still rare.

Recently, there were a number of focuses on existence problems, for example, see [9-11] and references therein, and functional differential equations were also investigated widely, such as work done in [12-14]. However, there were only very few results on existence of solutions for abstract measure functional differential equations.

There were some consideration on abstract measure delay differential equations [2] and perturbed abstract measure differential equations [4]. However, to the best of authors' knowledge, there were not any results dealing with perturbed abstract measure functional differential equations. In this article, we investigate the existence of solutions for perturbed abstract measure functional differential equations. This is a problem of difficulty and challenge. Based on the Leray-Schauder alternative involving the sum of two operators [15] and the Arzelà-Ascoli theorem, the existence results of our system is derived. The perturbed abstract measure functional differential system researched in this paper includes the systems studied in [2,4] as special cases. Additionally, considering appropriate degeneration, our sufficient conditions for existence of solutions are also less conservative than those in [2,4], respectively. The study in the previous articles are improved.

The content of this article is organized as follows: In Section 2, some preliminary fact is recalled; the perturbed abstract measure functional differential equation is proposed, as well as some relative notations. In Section 3, the existence theorem is given and strict proof is shown; two remarks are given to analyze that our existence results are less conservative. In Section 4, an example is used to illustrate the effectiveness of our results for existence of solutions.

2 Preliminary

Definition 2.1 Let X be a Banach space, a mapping $T : X \to X$ is called D-Lipschitzian, if there is a continuous and nondecreasing function $\varphi_T : \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$\| Tx - Ty \| \leq \phi_T(\| x - y \|)$$

for all $x, y \in X$, $\varphi_T(0) = 0$. T is called Lipschitzian, if $\varphi_T(x) = ax$, where $a > 0$ is a Lipschitz constant. Furthermore, T is called a contraction on X, if $a < 1$.

Let $T : X \to X$, where X is a Banach space. T is called totally bounded, if $T(M)$ is totally bounded for any bounded subset M of X. T is called completely continuous, if T is continuous and totally bounded on X. T is called compact, if $\overline{T(X)}$ is a compact subset of X. Every compact operator is a totally bounded operator.

Define any convenient norm $|| \cdot ||$ on X. Let x, y be two arbitrary points in X, then segment $\overline{xy}$ is defined as

$$\overline{xy} = \{z \in X \mid z = x + r(y - x),\ 0 \leq r \leq 1\}.$$

Let $x_0 \in X$ be a fixed point and $z \in X$, $\overline{0x_0} \subset \overline{0z}$, where 0 is the zero element of X. Then for any $x \in \overline{x_0 z}$, we define the sets S_x and $\bar{S}_x$ as

$$S_x = \{rx| - \infty < r < 1\},$$
$$\bar{S}_x = \{rx| - \infty < r \leq 1\}.$$

For any $x_1, x_2 \in \overline{x_0 z} \subset X$, we denote $x_1 < x_2$ if $S_{x_1} \subset S_{x_2}$, or equivalently $\overline{x_0 x_1} \subset \overline{x_0 x_2}$. Let $\omega \in [0, h]$, $h > 0$. For any $x \in \overline{x_0 z}$, x_ω is defined by

$$x_\omega < x,\ \| x - x_\omega \| = \omega.$$

Let M denote the σ-algebra which generated by all subsets of X, so that (X, M) is a measurable space. Let $ca(X, M)$ be the space consisting of all signed measures on M. The norm $|| \cdot ||$ on $ca(X, M)$ is defined as:

$$\| p \| = | p | (X),$$

where $|p|$ is a total variation measure of p,

$$| p | (X) = \sup_{\pi} \sum_{i=1}^{\infty} | p(E_i) |,\ E_i \subset X,$$

where $\pi : \{E_i : i \in \mathbb{N}\}$ is any partition of X. Then $ca(X, M)$ is a Banach space with the norm defined above.

Let μ be a σ-finite positive measure on X. $p \in ca(X, M)$ is called absolutely continuous with respect to the measure μ, if $\mu(E) = 0$ implies $p(E) = 0$ for some $E \in M$. And we denote $p \ll \mu$.

Let M_0 denote the σ-algebra on S_{x_0}. For $x_0 < z$, M_z denotes the σ-algebra on S_z. It is obvious that M_z contains M_0 and the sets of the form $S_x,\ x \in \overline{x_0 z}$.

Given a $p \in ca(X, M)$ with $p \ll \mu$, consider perturbed abstract measure functional differential equation:

$$\frac{dp}{d\mu} = f(x, p(\bar{S}_{x_\omega})) + g(x, p(\bar{S}_x), p(\bar{S}_{x_\omega})), \quad a.e. \ [\mu] \ on \ \overline{x_0 z}, \tag{1}$$

and

$$p(E) = q(E), \ E \in M_0. \tag{2}$$

where q is a given signed measure, $\frac{dp}{d\mu}$ is a Radon-Nikodym derivative of p with respect to μ. $f : S_z \times \mathbb{R} \to \mathbb{R}$, $g : S_z \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$. $f(x, p(\bar{S}_{x_\omega}))$ and $g(x, p(\bar{S}_x), p(\bar{S}_{x_\omega}))$ are μ-integrable for each $p \in ca(S_z, M_z)$.

Define

$$| f(x, p(\cdot)) | = \sup_{\omega \in [0,h]} | f(x, p(\bar{S}_{x_\omega})) |,$$
$$| g(x, p, p(\cdot)) | = \sup_{\omega \in [0,h]} | g(x, p(\bar{S}_x), p(\bar{S}_{x_\omega})) | .$$

Definition 2.2 q is a given signed measure on M_0. A signed measure $p \in ca(S_z, M_z)$ is called a solution of (1)-(2), if

(i) $p(E) = q(E)$, $E \in M_0$,

(ii) $p \ll \mu$ *on* $\overline{x_0 z}$,

(iii) p satisfies (1) a.e. $[\mu]$ on $\overline{x_0 z}$.

Remark 2.1 The system (1)-(2) is equivalent to the following perturbed abstract measure functional integral system:

$$p(E) = \begin{cases} \int_E f(x, p(\bar{S}_{x_\omega}))d\mu + \int_E g(x, p(\bar{S}_x), p(\bar{S}_{x_\omega}))d\mu, & E \in M_z,\ E \subset \overline{x_0 z} \\ q(E), & E \in M_0. \end{cases}$$

We denote a solution p of (1)-(2) as $p(\bar{S}_{x_0},\ q)$.

Definition 2.3 A function $\beta : S_z \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is called Carathéodory, if

(i) $x \to \beta(x, y, z)$ is μ-measurable for each $(y, z) \in \mathbb{R} \times \mathbb{R}$,

(ii) $(y, z) \to \beta(x, y, z)$ is continuous a.e. $[\mu]$ on $\overline{x_0 z}$.

The function β defined as the above is called L^1_μ-Carathéodory, further if

(iii) for each real number $r > 0$, there exists a function $h_r(x) \in L^1_\mu(S_z,\ \mathbb{R}^+)$ such that

$$| \beta(x,\ y,\ z) | \leq h_r(x) \ a.e. \ [\mu] \ on \ \overline{x_0 z}$$

for each $y \in \mathbb{R}$, $z \in \mathbb{R}$ with $|y| \leq r$, $|z| \leq r$.

Lemma 2.1 [15] Let $\mathscr{B}_r(0)$ and $\bar{\mathscr{B}}_r(0)$ denote, respectively, the open and closed balls in a Banach algebra X with center 0 and radius r for some real number $r > 0$. Suppose $A : X \to X$, $B :\ \bar{\mathscr{B}}_r(0) \to X$ are two operators satisfying the following conditions:

(a) A is a contraction, and

(b) B is completely continuous.

Then either

(i) the operator equation $Ax + Bx = x$ has a solution x in $\bar{\mathscr{B}}_r(0)$, or

(ii) there exists an element $u \in \partial\bar{\mathscr{B}}_r(0)$ such that $\lambda A(\frac{u}{\lambda}) + \lambda Bu = u$ for some $\lambda \in (0, 1)$.

3 Main results

We consider the following assumptions:

(A_0) For any $z \in X$ satisfies $x_0 < z$, the σ-algebra M_z is compact with respect to the topology generated by the pseudo-metric d defined by

$$d(E_1, E_2) = |\mu|(E_1 \Delta E_2), \quad E_1, E_2 \in M_z.$$

(A_1) $\mu(\{x_0\}) = 0$.

(A_2) q is continuous on M_z with respect to the pseudo-metric d defined in (A_0).

(A_3) There exists a μ-integrable function $\alpha : S_z \to \mathbb{R}^+$ such that

$$| f(x, y_1(\cdot)) - f(x, y_2(\cdot)) | \le \alpha(x) | y_1(\cdot) - y_2(\cdot) | \ \ a.e.[\mu] \text{ on } \overline{x_0 z}.$$

(A_4) $g(x, y, z(\cdot))$ is L^1_μ-Carath*é*odory.

Theorem 3.1 Suppose that the assumptions (A_0)-(A_4) hold. Further if $\| \alpha \|_{L^1_\mu} < 1$ and there exists a real number $r > 0$ such that

$$r > \frac{F_0 + \|q\| + \|h_r\|_{L^1_\mu}}{1 - \|\alpha\|_{L^1_\mu}} \tag{3}$$

where $F_0 = \int_{\overline{x_0 z}} | f(x, 0) | d\mu$. Then the system (1)-(2) has a solution on $\overline{x_0 z}$.

Proof: Consider the open ball $\mathscr{B}_r(0)$ and the closed ball $\bar{\mathscr{B}}_r(0)$ in $ca(S_z, M_z)$, with r satisfying the inequality (3). Define two operators $A : ca(S_z, M_z) \to ca(S_z, M_z)$, $B : \bar{\mathscr{B}}_r(0) \to ca(S_z, M_z)$ as:

$$Ap(E) = \begin{cases} \int_E f(x, p(\bar{S}_{x_\omega}))d\mu, & E \in M_z, E \subset \overline{x_0 z} \\ 0, & E \in M_0. \end{cases}$$

$$Bp(E) = \begin{cases} \int_E g(x, p(\bar{S}_x), p(\bar{S}_{x_\omega}))d\mu, & E \in M_z, E \subset \overline{x_0 z} \\ q(E), & E \in M_0. \end{cases}$$

Now we prove the operators A and B satisfy conditions that are given in Lemma 2.1 on $ca(S_z, M_z)$ and $\bar{\mathscr{B}}_r(0)$, respectively.

Step I. A is a contraction on $ca(S_z, M_z)$.

Let $p_1, p_2 \in ca(S_z, M_z)$. Then by assumption (A_3)

$$\begin{aligned} \left|Ap_1(E) - Ap_2(E)\right| &= \left| \int_E f(x, p_1(\bar{S}_{x_\omega}))d\mu - \int_E f(x, p_2(\bar{S}_{x_\omega}))d\mu \right| \\ &\le \int_E \alpha(x) \sup_\omega | p_1(\bar{S}_{x_\omega}) - p_2(\bar{S}_{x_\omega}) | d\mu \\ &\le \int_E \alpha(x) | p_1 - p_2 | (\bar{S}_x) d\mu \\ &\le \| \alpha \|_{L^1_\mu} | p_1 - p_2 | (E) \end{aligned}$$

for all $E \in M_z$.

Considering the definition of norm on $ca(S_z, M_z)$, we have

$$\| Ap_1 - Ap_2 \| \le \| \alpha \|_{L^1_\mu} \| p_1 - p_2 \|,$$

for all p_1, $p_2 \in ca(S_z, M_z)$. So A is a contraction on $ca(S_z, M_z)$.

Step II. B is continuous on $\bar{\mathscr{B}}_r(0)$.

Let $\{p_n\}_{n\in\mathbb{N}}$ be a sequence of signed measures in $\bar{\mathscr{B}}_r(0)$, and $\{p_n\}_{n\in\mathbb{N}}$ converges to a signed measure p. In case $E \in M_z$, $E \subset \overline{x_0 z}$, using dominated convergence theorem

$$\begin{aligned}\lim Bp_n(E) &= \lim_{n\to\infty}\int_E g(x, p_n(\bar{S}_x), p_n(\bar{S}_{x_\omega}))d\mu \\ &= \int_E g(x, p(\bar{S}_x), p(\bar{S}_{x_\omega}))d\mu \\ &= Bp(E).\end{aligned}$$

In case $E \in M_0$, $\lim\limits_{n\to\infty} Bp_n(E) = q(E) = Bp(E)$. Obviously, B is a continuous operator on $\bar{\mathscr{B}}_r(0)$.

Step III. B is a totally bounded operator on $\bar{\mathscr{B}}_r(0)$.

Let $\{p_n\}_{n\in\mathbb{N}}$ be a sequence of signed measures in $\bar{\mathscr{B}}_r(0)$, then $||p_n|| \le r(n \in \mathbb{N})$. Next we show that $\{Bp_n\}_{n\in\mathbb{N}}$ are uniformly bounded and equicontinuous.

First, $\{Bp_n\}_{n\in\mathbb{N}}$ are uniformly bounded. Let $E \in M_z$, and $E = F \cup G$, where $F \in M_0$ and $G \in M_z$, $G \subset \overline{x_0 z}$. $F \cap G = \emptyset$. Hence,

$$\begin{aligned}\left|Bp_n(E)\right| &\le | q(F) | + \int_G | g(x, p_n(\bar{S}_x), p_n(\bar{S}_{x_\omega})) | d\mu \\ &\le | q(F) | + \int_G h_r(x)d\mu,\end{aligned}$$

consequently,

$$\| Bp_n \| = | Bp_n | (S_z) = \sup \sum_{i=1}^{\infty} | Bp_n(E_i) | \le \| q \| + \| h_r \|_{L^1_\mu},$$

for every $p_n \in \bar{\mathscr{B}}_r(0)$. Then $\{Bp_n\}_{n\in\mathbb{N}}$ are uniformly bounded.

Second, $\{Bp_n\}_{n\in\mathbb{N}}$ is an equicontinuous sequence in $ca(S_z, M_z)$. Let $E_i \in M_z$, and $E_i = F_i \cup G_i$, where $F_i \in M_0$ and $G_i \in M_z$, $G_i \subset \overline{x_0 z}$, and $F_i \cap G_i = \emptyset$. $i = 1, 2$.

Considering assumption (A_4), then

$$\begin{aligned}|Bp_n(E_1) - Bp_n(E_2)| &\le |q(F_1) - q(F_2)| + |\int_{G_1} g(x, p_n(\bar{S}_x), p_n(\bar{S}_{x_\omega}))d\mu \\ &\quad - \int_{G_2} g(x, p_n(\bar{S}_x), p_n(\bar{S}_{x_\omega}))d\mu| \\ &\le |q(F_1) - q(F_2)| \\ &\quad + \int_{G_1 \Delta G_2} |g(x, p_n(\bar{S}_x), p_n(\bar{S}_{x_\omega}))|d\mu \\ &\le |q(F_1) - q(F_2)| + \int_{G_1 \Delta G_2} h_r(x)d\mu.\end{aligned}$$

when $d(E_1, E_2) \to 0$, $E_1 \to E_2$. Then, $F_1 \to F_2$, and $|\mu|(G_1 \Delta G_2) = d(G_1 \Delta G_2) \to 0$.

Considering assumption (A_2), q is continuous on compact M_z implies it is uniformly continuous on M_z. so

$$| Bp_n(E_1) - Bp_n(E_2) | \to 0, \ as \ d(E_1, \ E_2) \to 0$$

for every $p_n \in \bar{\mathscr{B}}_r(0)$.

$\{Bp_n\}_{n\in\mathbb{N}}$ is an equicontinuous sequence in $ca(S_z, M_z)$.

According to the Arzelà-Ascoli theorem, there is a subset $\{Bp_{n_k}\}_{n,k\in\mathbb{N}}$ of $\{Bp_n\}_{n\in\mathbb{N}}$ that converges uniformly. Thus, operator B is compact on $\bar{\mathscr{B}}_r(0)$. Then, B is a totally bounded operator on $\mathscr{B}_r(0)$.

From steps II and III, the operator B is completely continuous on $\mathscr{B}_r(0)$.

Step IV. (1)-(2) has a solution on $\overline{x_0 z}$.

Now, by applying Lemma 2.1, we show that (i) holds. Otherwise, there exists an element $u\in ca(S_z, M_z)$ with $||u|| = r$ such that $\lambda A(\frac{u}{\lambda}) + \lambda Bu = u$ for some $\lambda\in(0, 1)$.

If it is true, we have

$$u(E) = \begin{cases} \lambda \int_E f(x, \frac{u(\bar{S}_{x_\omega})}{\lambda})d\mu + \lambda\int_E g(x, u(\bar{S}_x), u(\bar{S}_{x_\omega}))d\mu, & E\in M_z, E\subset\overline{x_0 z} \\ \lambda q(E), & E\in M_0. \end{cases}$$

for some $\lambda\in(0, 1)$. Then

$$\begin{aligned} |u(E)| &\leq |\lambda A(\tfrac{u(E)}{\lambda})| + |\lambda B(u(E))| \\ &\leq \lambda|q(F)| + \lambda\int_G[|f(x, \tfrac{u(\bar{S}_{x_\omega})}{\lambda}) - f(x, 0)| + |f(x, 0)|]d\mu \\ &+ \lambda\int_G |g(x, u(\bar{S}_x), u(\bar{S}_{x_\omega}))|d\mu \\ &\leq |q(F)| + \int_G \alpha(x)|u(\bar{S}_{x_\omega})|d\mu + \int_G |f(x, 0)|d\mu + \int_G h_r(x)d\mu \\ &\leq |q(F)| + \|\alpha\|_{L^1_\mu}|u(E)| + \int_G |f(x, 0)|d\mu + \int_G h_r(x)d\mu. \end{aligned}$$

so we get

$$|u(E)| \leq \frac{|q(F)| + \int_G |f(x,0)|d\mu + \int_G h_r(x)d\mu}{1 - \|\alpha\|_{L^1_\mu}},$$

for all $E\in M_z$.

By the definition of the norm on $ca(S_z, M_z)$,

$$\|u\| \leq \frac{\|q\| + F_0 + \|h_r\|_{L^1_\mu}}{1 - \|\alpha\|_{L^1_\mu}}.$$

As $||u|| = r$, we have

$$r \leq \frac{\|q\| + F_0 + \|h_r\|_{L^1_\mu}}{1 - \|\alpha\|_{L^1_\mu}}.$$

This is a contradiction. Consequently, the equation $p(E) = Ap(E) + Bp(E)$ has a solution $p(\bar{S}_{x_0}, q)\in\bar{\mathscr{B}}_r(0)\subset ca(S_z, M_z)$. It is said that (1)-(2) has a solution on $\overline{x_0 z}$. The proof of Theorem 3.1 is completed.

Remark 3.1 If $f(x, y) = 0$ and ω is a given constant, then system (1)-(2) degenerates into

$$\frac{dp}{d\mu} = g(x, p(\bar{S}_x), p(\bar{S}_{x_\omega})), \ a.e.\ [\mu]\ on\ \overline{x_0 z}, \tag{4}$$

and

$$p(E) = q(E),\ E \in M_0, \tag{5}$$

obviously, (4)-(5) is the system (4) considered in [2]. Additionally, our degenerated assumptions for the existence theorem equal to (A_1)-(A_4) in [2], the more complex assumption (A_5) [2] is not necessary. So our results are less conservative.

Remark 3.2 If $\omega = 0$, then system (1)-(2) degenerates into

$$\frac{dp}{d\mu} = f(x, p(\bar{S}_x)) + g(x, p(\bar{S}_x)),\ a.e.\ [\mu]\ on\ \overline{x_0 z}, \tag{6}$$

and

$$p(E) = q(E),\ E \in M_0. \tag{7}$$

obviously, (6)-(7) is the system (3.6)-(3.7) studied in [4]. Additionally, our degenerated assumptions for the existence theorem equal to (A_0)-(A_2) and (B_0)-(B_1) in [4], the more complex assumption (B_2) [4] is not necessary. So, our results are less conservative.

4 Example

Let $p \in ca(S_z, M_z)$ with $p \ll \mu$. Consider the equation as follows:

$$\frac{dp}{d\mu} = \alpha(x) \mid p(\bar{S}_{x_\omega}) \mid + \frac{h_r(x)|p(\bar{S}_x) + p(\bar{S}_{x_\omega})|}{1 + |p(\bar{S}_x) + p(\bar{S}_{x_\omega})|},\ a.e.\ [\mu]\ \text{on}\ \overline{x_0 z}, \tag{8}$$

and

$$p(E) = q(E),\ E \in M_0. \tag{9}$$

where $h_r(x) \in L^1_\mu(S_z, \mathbb{R}^+)$, $\| \alpha \|_{L^1_\mu} < 1$ and $0 \le \omega \le h (h > 0)$. $f : S_z \times \mathbb{R} \to \mathbb{R}$ and $g : S_z \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are defined as

$$f(x,\ y(\cdot)) = \alpha(x) \mid p(\bar{S}_{x_\omega}) \mid,$$
$$g(x,\ y,\ z(\cdot)) = \frac{h_r(x) \mid p(\bar{S}_x) + p(\bar{S}_{x_\omega}) \mid}{1 + \mid p(\bar{S}_x) + p(\bar{S}_{x_\omega}) \mid}.$$

It is obvious that the assumptions (A_0) - (A_2) hold. Then, we show that f and g satisfy the assumptions (A_3) and (A_4), respectively.

First, f is continuous on $ca(S_z, M_z)$.

$$\begin{aligned} \left|f(x,\ y_1(\cdot)) - f(x,\ y_2(\cdot))\right| &\le \mid \alpha(x) \mid \sup_{\omega} \| p_1(\bar{S}_{x_\omega}) \mid - \mid p_2(\bar{S}_{x_\omega}) \| \\ &\le \mid \alpha(x) \mid \sup_{\omega} \mid p_1(\bar{S}_{x_\omega}) - p_2(\bar{S}_{x_\omega}) \mid \\ &= \mid \alpha(x) \mid \mid y_1(\cdot) - y_2(\cdot) \mid, \end{aligned}$$

$f(x, y(\cdot))$ satisfies (A_3).

Second, $|g(x, y, z(\cdot))| \le h_r(x)$. $g(x, y, z(\cdot))$ satisfies the assumption (A_4).

Thus, if there exists $r \in \mathbb{R}$ satisfies $r > \frac{F_0 + \|q\| + \|h_r\|_{L^1_\mu}}{1 - \|\alpha\|_{L^1_\mu}}$ with $F_0 = \int_{\overline{x_0 z}} \mid f(x,\ 0) \mid d\mu$, all conditions in

Theorem 3.1 are satisfied. So, (8)-(9) has a solution $p(\bar{S}_{x_0},\ q)$ on $\overline{x_0 z}$.

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Authors' contributions
JS directed the study and helped inspection. XW carried out the main results of this paper, including the existence theorem and the example. All the authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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